TOPOLOGY - III, EXERCISE SHEET 2

Exercise 1. $\Delta-$*complex as a quotient of simplices.* (medium)
Let X be a space with a $\Delta-$complex structure. Recall from the lecture that this means that there exists a collection of continuous maps $\{\sigma_\alpha : \Delta_\alpha^{n_\alpha} \to X\}_{\alpha \in I}$ for some indexing set I following certain properties. Recall also that edges of a simplex have the data of an orientation. Prove that X can be realised as a quotient of the disjoint union of simplices $\bigsqcup_{\alpha \in I} \Delta_\alpha^{n_\alpha}$ by identifying some faces of the same dimension and orientation.

Exercise 2. $\Delta-$*complexes are Hausdorff* (medium)
Show that if X is a $\Delta-$complex then it is a Hausdorff space.

Hint: Think of X as a quotient of $\bigsqcup_{\alpha \in I} \Delta_\alpha^{n_\alpha}$ and try to construct separating opens inductively.

Exercise 3. *Some surfaces as* $\Delta-$ *complexes.* (easy)
Show that the following spaces possess a $\Delta-$complex structure:

(1) $\mathbb{T}^2$.
(2) S^2.
(3) $\mathbb{RP}^2$.
(4) The Klein Bottle. (The surface one obtains by gluing two Möbius Strips along the equator).

Exercise 4. S^n *and* $\mathbb{RP}^n$ *as* $\Delta-$*complexes.* (medium)

(1) Show that S^n has a $\Delta-$complex structure by considering the polyhedron in $\mathbb{R}^{n+1}$ with $2n+2$ vertices; Two vertices on each coordinate axis.
(2) Realise $\mathbb{RP}^n$ as a $\Delta-$complex by using part (1).

Exercise 5. *Cone of a* $\Delta-$*complex.* (medium)
Let X be a topological space, we define the cone of X as the quotient:

$$CX := X \times [0, 1]/(X \times \{0\}).$$

Show that if X is a $\Delta-$complex then so is CX.